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CONFIDENCE ANALYSIS FOR FUZZY MULTI CRITERIA DECISION MAKING USING TRAPEZOIDAL FUZZY NUMBERS

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Multi criteria decision making (MCDM) deals with parameters which lack proper measurement scale, and are depicted using some linguistic parameters. In this paper, a general Fuzzy multi- criteria decision making problem (FMCDM) is introduced. Then confidence analysis of FMCDM is performed using trapezoidal fuzzy numbers by linguistic approach to model the decision maker's attitude. The approach is illustrated by a numerical example.

***Keywords*: Multi Criteria Decision Making, Linguistic Variable, Trapezoidal Fuzzy Numbers.

1. Introduction

Real world decision making problems are very often uncertain or vague in a number of ways. In many areas of daily life, human judgement, evaluation and decisions vary significantly based on individual's subjective perceptions. The decision maker's judgements cannot estimate his preference with an exact numerical value. Hence crisp data is inadequate to model real life situations. A more realistic approach is required to use the linguistic assessments instead of numerical values. These characteristics indicate that fuzzy theory can be applied effectively to deal with such problems.

Multi criteria decision-making (MCDM) refers to screening, prioritizing, ranking or selecting the alternatives based on human judgement. Classical MCDM methods cannot handle problems with imprecise and incomplete data. The most appropriate tool to tackle such problems is the application of fuzzy theory in those problems.

The fundamental concept of fuzzy theory is that any field *X* and theory *Y* can be fuzzified by replacing the concept of a crisp set in *X* and *Y* by that of a fuzzy set. Mathematically a fuzzy set can be defined by assigning to each possible individual in the universe of discourse, a value representing its grade of membership in the fuzzy set. The membership function denoted by μ is defined from *X* to [0, 1].

1.1. Definitions and Formulations:

Definition 1.1.1: Linguistic Variable

A linguistic variable is a variable whose values are linguistic terms. The concept of linguistic variable is applied in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions.

For example, 'height; is a linguistic variable, its values can be very high, high, medium, low, very low etc., These values can also be represented by fuzzy numbers.

Definition 1.1.2: Triangular Fuzzy Number

A triangular fuzzy number $\hat{a}$ is defined by a triplet (a_1, a_2, a_3). The membership function is defined as

$$\mu_{\hat{a}}(x) = \begin{cases} (x-a_1)/(a_2-a_1) & \text{if } a_1 \le x \le a_2 \\ (a_3-x)/(a_3-a_2) & \text{if } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

The triangular fuzzy number is based on three-value judgement: The minimum possible value a_1, the most possible value a_2 and the maximum possible value a_3.

Definition 1.1.3: Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\hat{a}$ is a fuzzy number (a_1, a_2, a_3, a_4) and its membership function is defined as

$$\mu_{\hat{a}}(x) = \begin{cases} (x-a_1)/(a_2-a_1) & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } a_2 \le x \le a_3 \\ (x-a_4)/(a_3-a_4) & \text{if } a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases}$$

From the definition of triangular and trapezoidal fuzzy numbers it is clear that the triangular fuzzy number is a special case of trapezoidal number.

Trapezoidal fuzzy numbers are more suitable than triangular fuzzy numbers in some cases. For example consider the linguistic term 'average production' of a company. The linguistic term average production may be

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represented as (40, 50, 60) (in tonnes) using triangular fuzzy numbers. From this we can say that average production of the company is 50 tonnes. But the same can be represented as (40, 50, 55, 60) using trapezoidal fuzzy numbers. From this we say that the average production of the company ranges from 50 to 55 tonnes. The advantage of using trapezoidal fuzzy number is that it attains membership value 1 between two points (i.e.) in an interval whereas the triangular fuzzy number attains the same at only one point. Hence trapezoidal fuzzy numbers are more suitable than triangular fuzzy numbers in problems involving linguistic terms.

1.2. A General Fuzzy MCDM

A general MCDM problem with 'm' alternatives A_i (i = 1, 2, …… m) and 'n' criteria C_j (j = 1, 2, 3 ….. n) can be expressed as

$$D = [x_{ij}] \text{ and } W = [w_j] \text{ where } i = 1, 2, \ldots m \text{ and } j = 1, 2, \ldots n \quad (1.2.1)$$

Here D is the decision matrix; x_{ij} represents the rating of each alternative A_i with respect to each criterion C_j; W represents the weight vector and w_j represents the weight of criterion C_j.

Generally criteria are classified into two types;

- Benefit criteria and
- Cost criteria.

The decision maker expects a value as high as possible for benefit criteria and a value as low as possible for cost criteria.

In a fuzzy environment,

$$\mathbf{\tilde{D} = [\tilde{x}_{ij}] \text{ and } \tilde{W} = [\tilde{w}_j] \text{ where}} \quad (1.2.2)$$

$\tilde{x}_{ij}$ represents the fuzzy rating of each alternative A_i with respect to each criterion C_j; $\tilde{w}_j$ represents the fuzzy weight of criterion C_j.

Normalisation

The fuzzy numbers in the decision matrix is normalized in to the performance matrix $\tilde{p} = [\tilde{p}_{ij}]$, where

$$\left[\tilde{p}_{ij}\right] = \begin{cases} [x_{ij1}/B, x_{ij2}/B, x_{ij3}/B, x_{ij4}/B & B = \max_i x_{ij}, C_j \text{ is Benefit criterion,} \\ [(C - x_{ij4})/C, (C - x_{ij3})/C, (C - x_{ij2}/C, (C - x_{ij1}/C & C = \max_i x_{ij}, C_j \text{ is Cost criterion} \end{cases} \quad (1.2.3)$$

The normalisation method mentioned above is to preserve the property that the ranges of normalized trapezoidal fuzzy numbers belong to [0, 1].

Weighting the Criteria

The weighted performance matrix is constructed by multiplying the weight vector with the decision matrix.

$$\tilde{P}^w \left[\tilde{p}_{ij}{}^w\right], \quad (1.2.4)$$

$$\tilde{p}_{ij1}{}^w = w_{j1}\, p_{ij1}, \tilde{p}_{ij2}{}^w = w_{j2}\, p_{ij2}, \tilde{p}_{ij3}{}^w = w_{j3}\, p_{ij3}, \tilde{p}_{ij4}{}^w = w_{j4}\, p_{ij4}, i = 1, 2 \ldots m,\ j = 1, 2 \ldots n.$$

Performance of Alternatives

The vertex method [2] is used to calculate the alternative's performance index with respect to ideal solutions. It is based upon the concept that the chosen alternative should have the shortest distance from the Positive ideal solution (PIS) i.e. the solution that maximizes the benefit criteria and minimizes the cost criteria; and the farthest distance from the Negative ideal solution (NIS), i.e. the solution that maximizes the cost criteria and minimizes the benefit criteria.

For the normalized fuzzy performance matrix, the PIS is defined as

$\tilde{p}_j{}^* = (1, 1, 1, 1)$ and the NIS is defined as

$$\tilde{p}_j{}^- = (0, 0, 0, 0). \quad (1.2.5)$$

By the vertex method,

$$d_i^* = \sum_{j=1}^{n} d\left(\tilde{p}_{ij}^w, \tilde{p}_j^*\right) \quad (1.2.6)$$

$$d_i^- = \sum_{j=1}^{n} d\left(\tilde{p}_{ij}^w, \tilde{p}_j^-\right) \quad (1.2.7)$$

where i = 1, 2, …. m, j = 1, 2, ….. n.

d_i^* and d_i^- are the distance between each alternative and the PIS and NIS respectively.

The performance of each alternative is calculated using

$$p_i = [d_i^- + n - d_i^*] / 2n, \; i = 1, 2, \ldots m \quad (1.2.8)$$

where 'n' is the number of the criteria.

The nearer p_i gets to 1, the better the alternative's performance.

1.3 Example

Suppose that a person wants to buy a Lap- top manufactured by one of the top most companies. The MCDM problem is to select a model from three alternatives namely

- Acer
- Sony

- Toshiba

The criteria considered are

- Price
- Processor speed
- Weight and
- Reliability

Among these Processor speed is a benefit criterion. The criteria price and weight are cost criteria and are measured in thousands of Rupees and pounds (lb) respectively. The criterion reliability is a value criterion measured on a convenient scale from 0 to 10.

Table 1 gives the decision matrix and the weight for the corresponding criteria. The weights for the criteria are chosen such that their sum is 1 to simplify calculations. The weights are indicated in the brackets.

In the decision matrix the ratings are expressed as trapezoidal fuzzy numbers. For example, the third alternative Toshiba's price ranges from 26.3 to 26.4 and can be as low as 26.1 and as high as 26.5.

Table 1

Criteria → *Alternatives ↓*	*Price (0.4)* *(Thousands of Rupees)*	*Processor speed (0.2)* *(Hundreds of MHz)*	*Weight (0.2)* *(lbs)*	*Reliability (0.2)* *(1-10)*
Acer	(27,27.3, 27.4.27.5)	(13,13.5, 14,15)	(5.5,5.5, 5.5,5.5)	(4,6,7,8)
Sony	(28,28.25, 28.4,28.5)	(13.5,14, 14.5,15)	(6.2,6.2, 6.2,6.2)	(7,7.5, 8,10)
Toshiba	(26.1,26.3, 26.4,26.5)	(30,31, 32,33)	(9.5,9.5, 9.5,9.5)	(1,2,3.5,4)

The performance matrix for the decision matrix of Table 1 is calculated using the formula given in (1.2.3). The results are shown in Table 2.

Table 2

Criteria → *Alternatives ↓*	*Price (0.4)* *(Thousands of Rupees)*	*Processor speed (0.2)* *(Hundreds of MHz)*	*Weight (0.2)* *(lbs)*	*Reliability (0.2)* *(1-10)*
Acer	(0.035, 0.0386, (0.0421, 0.0526)	(0.3939, 0.409, 0.4242, 0.4545)	(0.421, 0.421, 0.421, 0.421)	(0.4,0.6, 0.7,0.8)
Sony	(0, 0.0035, 0.0088, 0.0175)	(0.409, 0.4242, 0.4394, 0.4545)	(0.3474, 0.3474, 0.3474, 0.3474)	(0.7, 0.75, 0.8,1)
Toshiba	(0.07, 0.0737, 0.0772, 0.0842)	(0.909, 0.9394, 0.9697, 1)	(0, 0, 0, 0)	(0.1,0.2, 0.35,0.4)

The Weighted performance matrix for the matrix given in Table 2 is constructed using (1.2.4) and shown in Table 3.

Table 3

Criteria → *Alternatives ↓*	*Price (0.4)* *(Thousands of Rupees)*	*Processor speed (0.2)* *(Hundreds of MHz)*	*Weight (0.2)* *(lbs)*	*Reliability (0.2)* *(1-10)*
Acer	(0.140, 0.1544, 0.01684, 0.02104)	(0.07878, 0.0818, 0.08484, 0.090)	(0.0842, 0.0842, 0.0842, 0.0842)	(0.08, 0.12, 0.14, 0.16)
Sony	(0, 0.0014, 0.00352, 0.007)	(0.0818, 0.08484, 0.08788, 0.0909)	(0.06948, 0.06948, 0.06948, 0.06948)	(0.14, 0.15, 0.16, 0.2)
Toshiba	(0.028, 0.02948, 0.03088, 0.03368)	(0.1818, 0.18788, 0.19394, 0.2)	(0, 0, 0, 0)	(0.02, 0.04, 0.07, 0.08)

The performance index for the alternatives is calculated using (1.2.8) and the results are tabulated in Table 4.

Table 4

Alternatives	*Performance index*	*Rank*
Acer	0.0779	3
Sony	0.0806	2
Toshiba	0.0848	1

Thus the decision maker ranks Toshiba as the first, Sony as the second and Acer as the third.

2. CONFIDENCE ANALYSIS OF FMCDM

The method described above does not provide the decision makers confidence about the rating. For e.g. A person who is extremely confident about the rating would believe that the most likely value is true in each case. But a person who is extremely about non-confident about the ratings will not do so. Hence it is important to incorporate the decision maker's confidence about the ratings. To accomplish the confidence about a Trapezoidal fuzzy number we adopt the α-cut concept followed in [5].

[5] uses $\alpha \in [0, 1]$ as a basic measure of confidence about the fuzzy number. This is used to compute a refined fuzzy number that is "closer" to the value with highest possibility as α tends to 1.

Assuming that the confidence in the Trapezoidal fuzzy numbers $\bar{a} = (a_1, a_2, a_3, a_4)$ is at level α, the refined fuzzy number is defined as

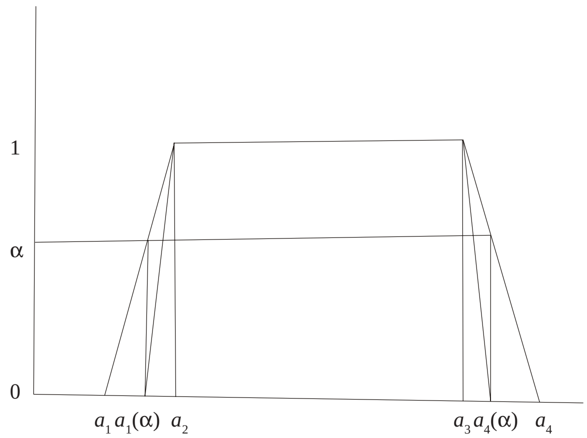
$$\bar{a}^{\alpha} = [a_1(\alpha), a_2, a_3, a_4(\alpha)]$$
$$= [a_1 + \alpha(a_2 - a_1), a_2, a_3, a_4 - \alpha(a_4 - a_3)] \quad (2.1)$$

If there are "L" confidence levels, then the confidence level "α" is obtained as

$$\alpha = (k-1)/(L-1),\ L \geq 2 \qquad (2.2)$$

$i = 1, 2, 3, \ldots m$ and $k = 1, 2, 3, \ldots L$.

We use a linguistic variable to represent the decision maker's confidence about the rating. For that we use a nine point linguistic scale as given in the following Table.



We take the confidence level $\alpha = 1$ for the linguistic term extremely confident and $\alpha = 0$ for the term extremely non confident respectively. Since there are 9 confidence levels, the other α value are calculated from 2.2. Hence we have the following Table.

The decision matrix with given confidence level α is constructed as $\tilde{\mathbf{D}}^{\alpha} = [\tilde{\mathbf{x}}_{ij}^{\alpha}]$.

Table 6

Linguistic Term	*Confidence Level* α
Extremely confident	1
Very confident	0.875
Confident	0.75
Fairly confident	0.625
Neutral	0.5
Fairly non confident	0.375
Non confident	0.25
Very non confident	0.125
Extremely non confident	0

Where $\tilde{\mathbf{x}}_{ij}^{\alpha}$ is the trapezoidal fuzzy number derived from $\tilde{\mathbf{x}}_{ij}$ under the specific confidence level by (2.1).

As described in the previous section the normalization and weighting process are completed and the performance index is calculated. The following results are obtained.

Hence a decision maker ranging from extremely confident to non confident ranks Sony as the first, Acer as the second and Toshiba as the third. But a person who is very non confident ranks Acer as the first, Toshiba as the second and Sony as the third and an extremely non confident person ranks Toshiba as the first, Sony as the second and Acer as the third alternative.

Table 7

Linguistic Term	*Acer*		*Sony*		*Toshiba*	
	Performance index	*Rank*	*Performance index*	*Rank*	*Performance index*	*Rank*
Extremely confident	0.0869	2	0.0884	1	0.0739	3
Very confident	0.0846	2	0.0873	1	0.0740	3
Confident	0.0829	2	0.0857	1	0.0711	3
Fairly confident	0.0830	2	0.0852	1	0.0723	3
Neutral	0.0828	2	0.0840	1	0.0708	3
Fairly non confident	0.0805	2	0.0832	1	0.0702	3
Non confident	0.0795	2	0.0822	1	0.0701	3
Very non confident	0.0881	1	0.0813	3	0.0878	2
Extremely non confident	0.0779	3	0.0806	2	0.0848	1

4. CONCLUSION

Since crisp data is inadequate in dealing real life situations in MCDM, it is important to incorporate the uncertainty prevailing in the problem. We have presented a FMCDM approach based on Decision Maker's confidence and we hope that the approach is effective in dealing with such problems.

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